settle. Intuition and comparison with the wall-shear singularity suggests that the free-stagnation-point flow will be singular only in special quasisteady cases where the longitudinal acceleration remains finite.

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On the Transonic-Dip Mechanism of Flutter of a Sweptback Wing

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Nomenclature

a	= distance of elastic axis behind midchord, percent semichord
b	= semichord
g	= structural damping coefficient
h	= vertical displacement of elastic axis
K_α	= spring constant of pitching oscillation
K_h	= spring constant of bending oscillation
L	= lift per unit span
M	= Mach number
M_α	= pitching moment about elastic axis per unit span
m	= mass of airfoil per unit span
$r_{c.g.}$	= nondimensional radius of gyration about center of gravity
V_F	= flutter velocity
$x_{c.g.}$	= distance of center of gravity behind midchord, percent semichord
x_{pv}	= distance of pivotal point behind midchord, percent semichord
α	= pitching displacement
μ	= mass ratio, $m/\pi\rho b^2$
ρ	= air density of freestream
$\phi_{h,\alpha}$	= phase difference between bending and torsional oscillation, bending leading the torsional motion
ω_h, ω_α	= uncoupled circular frequency of the airfoil in bending and in pitch, respectively
ω_1, ω_2	= first and second natural circular frequency of airfoil
ω_F	= circular frequency at flutter

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Introduction

As is well known, the transonic flight region presents critical conditions for the flutter-free requirement of various wing surfaces. This situation is especially severe for sweptback wings since they experience a sharp drop of the flutter boundary (transonic dip).[1,2]

As pointed out by Mykytow,[1] there is a detrimental effect of mass ratio on the flutter boundary in the transonic region; the greater the mass ratio, the deeper the dip. In the experimental data obtained by Farmer and Hanson[2] for a sweptback wing of large aspect ratio, the flutter frequency decreases from 18 Hz at $M=0.6$ to around 10 Hz near $M=1.0$, where there is a sharp transonic dip. These characteristics of sweptback wing flutter in the transonic region suggest a mechanism of the transonic dip phenomenon. The purpose of the present study is to investigate this mechanism and identify the essential structural and unsteady aerodynamic features.

Analysis

As pointed out by Cunningham[3] in his paper on pure-bending flutter of a sweptback wing, the first bending mode of the sweptback wing can absorb energy from the airstream. The fundamental mechanism of this can be explained as follows. When we look at the streamwise sections, we notice that they pivot around the axes near or ahead of the leading edge. In this case, the work per cycle done by the aerodynamic pitching moment (about the pivotal point) on the airfoil motion becomes positive if there is a time lag between the airfoil motion and the pitching moment.[4] This destabilizing time lag is caused by the effects of the shed vortices[4] and that of the flow compressibility.[3,5] Since the latter compressibility effect on the time lag is most pronounced in the transonic region, it is quite possible that this mechanism of pure-bending flutter might be dominating the transonic dip phenomenon. The characteristic behavior of sweptback wing flutter such as the strong dependence on the mass ratio and the decrease of flutter frequency in the transonic dip also seem to support this idea. To confirm this, we made some flutter calculations of a two-dimensional wing, which has a similar vibrational characteristic to a streamwise section of a typical sweptback wing.

As already pointed out, the streamwise sections of a sweptback wing in the first natural vibration mode have pivotal points ahead of the leading edge. A close examination of the first natural modes of the flutter models reported in Ref. 6 (the sweptback series), which show a sharp transonic dip of the flutter boundary, has revealed that the pivotal points are located from one to two chords ahead of the

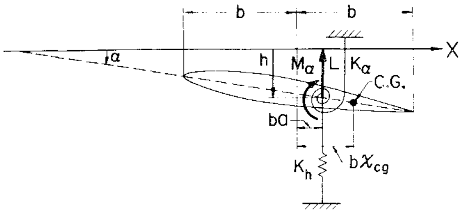

Fig. 1 Binary system of airfoil.

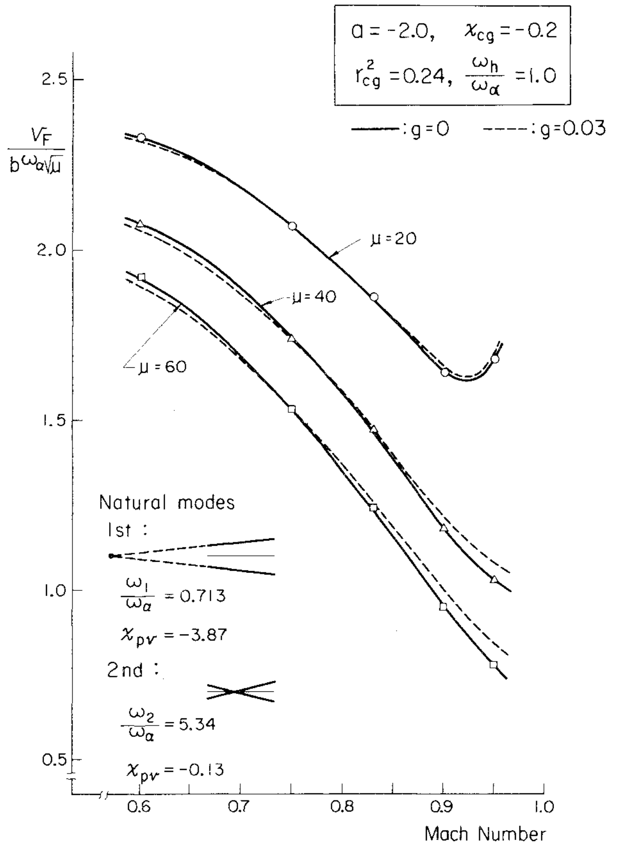

Fig. 2a Natural vibration modes and flutter velocity coefficient as functions of Mach number ($g = 0$ and $g = 0.03$).

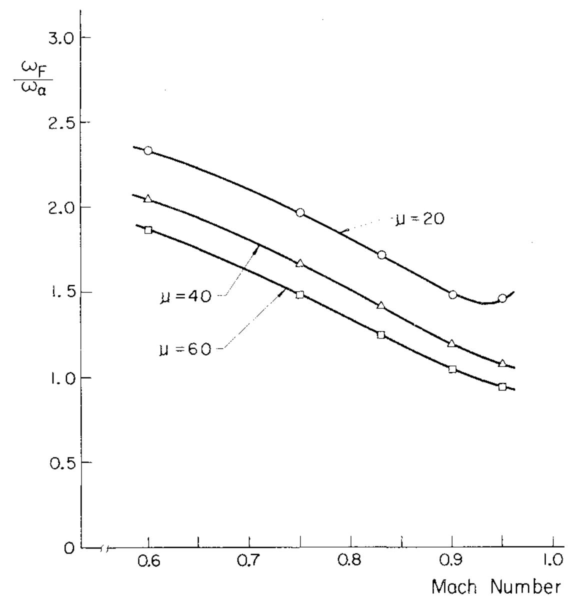

Fig. 2b Flutter frequency as function of Mach number ($g = 0$).

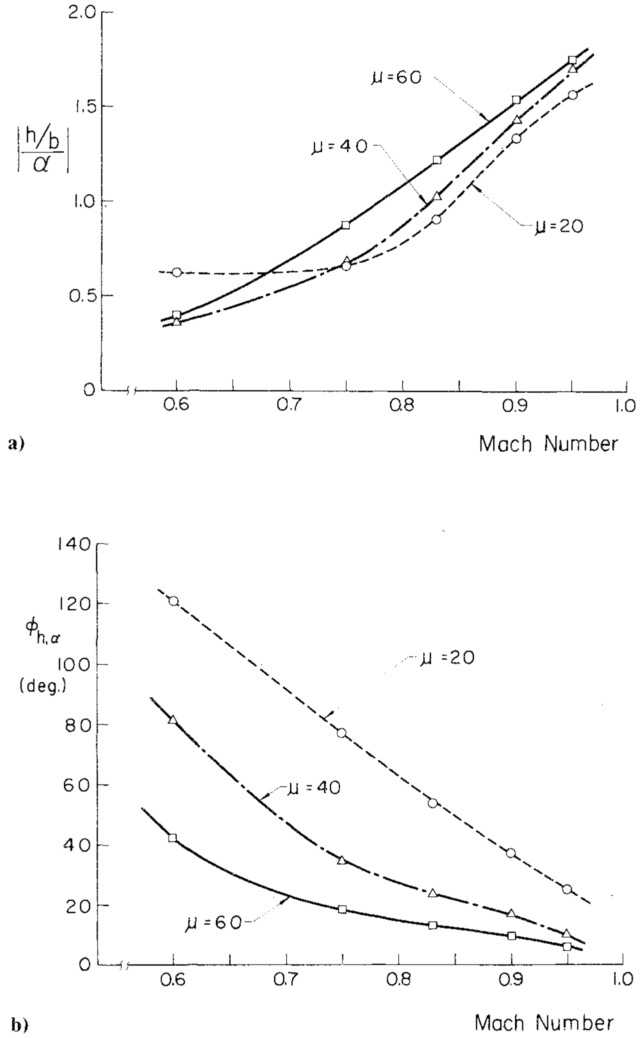

Fig. 3 Flutter modes as functions of Mach number ($g = 0$), a) amplitude ratio; b) phase difference.

leading edge for the midspan to tip sections. Another characteristic is the relatively high ratio of the first torsional frequency to the first bending frequency, ranging from six to seven. The behavior of the streamwise sections having such vibrational characteristics can be well simulated by an ordinary binary (bending-torsion) system of a two-dimensional airfoil shown in Fig. 1[7] by carefully choosing the values of the structural parameters. Placing $a = -2.0$ and $\omega_h/\omega_\alpha = 1.0$, we obtain the natural modes shown in Fig. 2a that closely simulate the above-mentioned characteristic behaviors of the streamwise sections of a sweptback wing. The values of $r^2_{c.g.}$ and $x_{c.g.}$ are taken from one of the examples of Ref. 8. The flutter calculations have been performed with this model, using the linearized subsonic unsteady aerodynamic forces,[9] for various Mach numbers ($M = 0.60$-0.95) with three different values of mass ratio μ, namely $\mu = 20$, 40, and 60, which are well in the range encountered by modern aircraft.

Results and Discussion

In Figs. 2a and 2b, the nondimensional flutter velocity coefficient $V_F/(b\omega_\alpha\sqrt{\mu})$ and the corresponding flutter frequency ω_F/ω_α are plotted against each Mach number, respectively. The similar characteristics of the flutter boundary as observed for the sweptback wings[1] is seen in Fig. 2a. Namely, there is a detrimental effect of the mass ratio, especially for $M = 0.90$-0.95. The dip is deeper for larger mass ratios. (As seen in Fig. 2a, the effect of the structural damping is relatively small for this range of the mass ratio.) The considerable decrease of the flutter frequency with increasing Mach number is also seen in Fig. 2b. In Figs. 3a and 3b, the corresponding flutter modes are plotted. The remarkable feature of these flutter modes is that the phase difference ($\phi_{h,\alpha}$) between h motion and α motion rapidly decreases with increasing Mach number and the mass ratio. This implies an interesting fact as to the mechanism of the flutter. The large phase difference ($\phi_{h,\alpha}$) between the h motion and α motion at $M=0.6$ indicates that the flutter at this Mach number is essentially a classical-type (bending-torsion) flutter for which the phase difference between the degrees of freedom is playing the dominant role.[10,11] On the other hand, the phase difference ($\phi_{h,\alpha}$) at $M=0.90$-0.95 tends to approach zero as the mass ratio increases. (This tendency is further confirmed by calculating the more extreme case of $\mu=300$ ($M=0.95$), for which $\phi_{h,\alpha}$ becomes less than 1 deg.) Since the zero phase difference between the h motion and the α motion indicates the existence of the pivotal point at $x_{pv}=a-(h/b)/\alpha$,[11] the flutter mode, for instance, at $M=0.95$ and $\mu=60$ is essentially a pitching oscillation with the pivotal point at $x_{pv}=-3.75$ (if we neglect the phase difference of 5.6 deg), which is very close to the first natural mode shown in Fig. 2a. From these facts, we can derive an interesting conclusion that the mechanism of the single-degree-of-freedom flutter is dominating the flutter of this system at the bottom of the transonic dip. It is also evident from the mechanism of single-degree-of-freedom flutter as discussed in the previous section that the large time lag between the aerodynamic pressures and the airfoil motion in the transonic region, which is caused by the compressibility effect, is the main cause of the transonic dip phenomenon.

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Eigenrelations for General Second-Order Systems

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THE motion of a typical n degree of freedom second-order linear system is governed by the matrix differential equation

$$[M]\ddot{x}+[C]\dot{x}+[K]x=F \tag{1}$$

where the $[M]$, $[C]$, and $[K]$ matrices are constant coefficient and generally nonsymmetric. This equation is usually solved in state space[1,2] due to the availability of first-order eigenvalue-vector subroutines. An eigenvector expansion solution procedure can, however, be applied for either the first-order state space representation or for the original second-order representation of Eq. (1).[3-5] Both procedures are outlined in this Note and are shown to be equivalent.

The state space representation of the system is

$$[A]\dot{y}+[B]y=P \tag{2}$$

in which

$$y=\begin{Bmatrix}\dot{x}\\ x\end{Bmatrix} \tag{3}$$

$$[A]=\begin{bmatrix}[0] & [M]\\ [M] & [C]\end{bmatrix} \tag{4}$$

$$[B]=\begin{bmatrix}[-M] & [0]\\ [0] & [K]\end{bmatrix} \tag{5}$$

$$P=\begin{Bmatrix}0\\ F\end{Bmatrix} \tag{6}$$

The complex eigenvalues and right vectors from the homogeneous form of Eq. (2) satisfy the algebraic eigenvalue problem

$$(\lambda_i[A]+[B])y_i=0 \tag{7a}$$

or equivalently

$$[D]y_i=(1/\lambda_i)y_i \tag{7b}$$

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